# Galindo-Garcia Identity-Based Signature Revisited.

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# FORMAL DEFINITIONS

# Definition–Public-Key Signature

An PKS scheme consists of three PPT algorithms $\{\mathcal{K}, \mathcal{S}, \mathcal{V}\}$

- Key Generation, $\mathcal{K}$
  - Used by the user to generate the public-private key pair $(\mathtt{pk}, \mathtt{sk})$
  - $\mathtt{pk}$ is published and the $\mathtt{sk}$ kept secret
  - Run on a *security parameter* $\kappa$

$$(\mathtt{pk}, \mathtt{sk}) \xleftarrow{\$} \mathcal{K}(\kappa)$$

- Signing, $\mathcal{S}$
  - Used by the user to generate signature on some message $m$
  - The secret key $\mathtt{sk}$ used for signing

$$\sigma \xleftarrow{\$} \mathcal{S}(\mathtt{sk}, m)$$

- Verification, $\mathcal{V}$
  - Outputs 1 if $\sigma$ is a valid signature on $m$; else, outputs 0

$$\mathrm{b} \leftarrow \mathcal{V}(\sigma, m, \mathtt{pk})$$

# Definition–Identity-Based Signature

An IBS scheme consists of four PPT algorithms $\{\mathcal{G}, \mathcal{E}, \mathcal{S}, \mathcal{V}\}$

- Set-up, $\mathcal{G}$
  - Used by the PKG to generate the public parameters ($\texttt{mpk}$) and master secret ($\texttt{msk}$)
  - $\texttt{mpk}$ is published and the $\texttt{msk}$ kept secret
  - Run on a *security parameter* $\kappa$

$$(\texttt{mpk}, \texttt{msk}) \stackrel{\$}{\leftarrow} \mathcal{G}(\kappa)$$

- Key Extraction, $\mathcal{E}$
  - Used by the PKG to generate the user secret key ($\texttt{usk}$)
  - $\texttt{usk}$ is then distributed through a secure channel

$$\texttt{usk} \stackrel{\$}{\leftarrow} \mathcal{E}(\texttt{id}, \texttt{msk})$$

# Definition–Identity-Based Signature...

An IBS scheme consists of four PPT algorithms $\{\mathcal{G}, \mathcal{E}, \mathcal{S}, \mathcal{V}\}$

- Signing, $\mathcal{S}$
  - Used by a user with identity `id` to generate signature on some message $m$
  - The user secret key `usk` used for signing

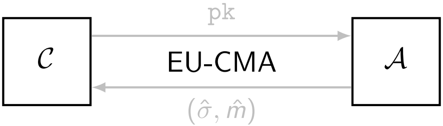
$$\sigma \xleftarrow{\$} \mathcal{S}(\mathtt{usk}, \mathtt{id}, m, \mathtt{mpk})$$

- Verification, $\mathcal{V}$
  - Outputs 1 if $\sigma$ is a valid signature on $m$ by the user with identity `id`
  - Otherwise, outputs 0

$$\mathrm{b} \leftarrow \mathcal{V}(\sigma, \mathtt{id}, m, \mathtt{mpk})$$

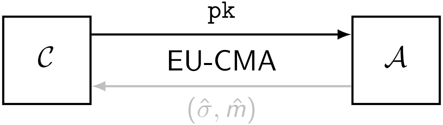
# SECURITY MODELS FOR PKS AND IBS

# Security Model for PKS–EU-CMA




- Existential unforgeability under chosen-message attack

# Security Model for PKS–EU-CMA




- Existential unforgeability under chosen-message attack
- $\mathcal{C}$ generates key-pair $(\mathtt{pk}, \mathtt{sk})$ and passes $\mathtt{pk}$ to $\mathcal{A}$.

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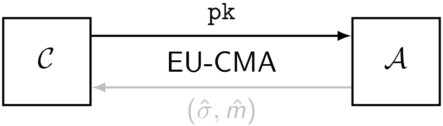


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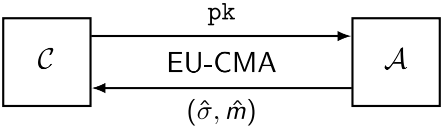
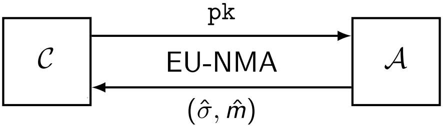



- Existential unforgeability under chosen-message attack
- $\mathcal{C}$ generates key-pair $(\mathtt{pk}, \mathtt{sk})$ and passes $\mathtt{pk}$ to $\mathcal{A}$.
- Signature Queries: Access to a signing oracle
- Forgery: $\mathcal{A}$ wins if
  - $\hat{\sigma}$ is a *valid* signature on $\hat{m}$.
  - $\mathcal{A}$ has *not* made a signature query on $\hat{m}$.
- Adversary's advantage in the game:

$$\Pr\left[1 \leftarrow \mathcal{V}(\hat{\sigma}, \hat{m}, \mathtt{pk}) \mid (\mathtt{sk}, \mathtt{pk}) \xleftarrow{\$} \mathcal{K}(\kappa); (\hat{\sigma}, \hat{m}) \xleftarrow{\$} \mathcal{A}(\mathtt{pk})\right]$$

# Security Model for PKS–EU-NMA




- Existential unforgeability under no-message attack
- $\mathcal{C}$ generates key-pair $(\mathtt{pk}, \mathtt{sk})$ and passes $\mathtt{pk}$ to $\mathcal{A}$.
- ~~Signature Queries: Access to a signing oracle~~
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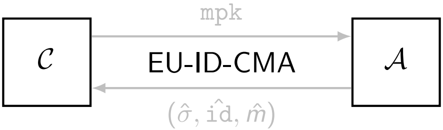
# Security Model for IBS: EU-ID-CMA




- Existential unforgeability with adaptive identity under no-message attack

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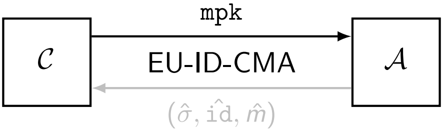


- Existential unforgeability with adaptive identity under no-message attack
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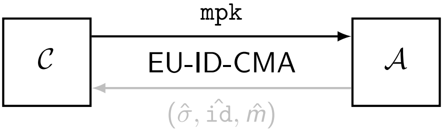


- Existential unforgeability with adaptive identity under no-message attack
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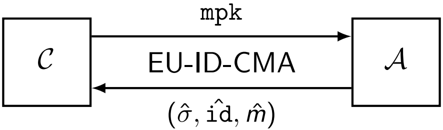
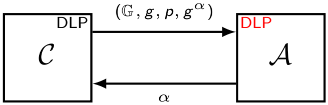



- Existential unforgeability with adaptive identity under no-message attack
- $\mathcal{C}$ generates key-pair $(\mathtt{mpk}, \mathtt{msk})$ and passes $\mathtt{mpk}$ to $\mathcal{A}$.
- Extract Queries, Signature Queries
- Forgery: $\mathcal{A}$ wins if
  - $\hat{\sigma}$ is a *valid* signature on $\hat{m}$ by $\hat{\mathtt{id}}$.
  - $\mathcal{A}$ has *not* made an extract query on $\hat{\mathtt{id}}$.
  - $\mathcal{A}$ has *not* made a signature query on $(\hat{\mathtt{id}}, \hat{m})$.
- Adversary's advantage in the game:

$$\Pr\left[1 \leftarrow \mathcal{V}(\hat{\sigma}, \hat{\mathtt{id}}, \hat{m}, \mathtt{mpk}) \mid (\mathtt{msk}, \mathtt{mpk}) \xleftarrow{\$} \mathcal{G}(\kappa); (\hat{\sigma}, \hat{\mathtt{id}}, \hat{m}) \xleftarrow{\$} \mathcal{A}(\mathtt{mpk})\right]$$

# Hardness Assumption: Discrete-log Assumption

Discrete-log problem for a group $\mathbb{G} = \langle g \rangle$ and $|\mathbb{G}| = p$



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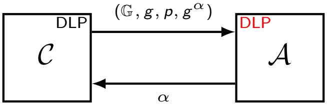



Definition. The DLP in $\mathbb{G}$ is to find $\alpha$ given $g^{\alpha}$, where $\alpha\mathbb{Z}_p$. An adversary $\mathcal{A}$ has advantage $\epsilon$ in solving the DLP if

$$\Pr\left[\alpha' = \alpha \mid \alpha\mathbb{Z}_p; \alpha' \leftarrow \mathcal{A}(\mathbb{G}, p, g, g^{\alpha})\right] \geq \epsilon.$$

The $(\epsilon, t)$-discrete-log assumption *holds* in $\mathbb{G}$ if no adversary has advantage at least $\epsilon$ in solving the DLP in time at most $t$.

# GALINDO-GARCIA IBS

# Galindo-Garcia IBS - Salient Features

- Derived from Schnorr signature scheme
- Based on the *discrete-log* assumption
- Efficient, simple and does not use *pairing*
- Security argued using *oracle replay* attacks
- Uses the *random oracle* heuristic

# SCHNORR SIGNATURE AND THE ORACLE REPLAY ATTACK

# Schnorr Signature

*The Setting.*

1. We work in group $\mathbb{G} = \langle g \rangle$ of prime order $p$.
2. A hash function $\mathsf{H} : \{0,1\}^* \to \mathbb{Z}_p$ is used.

*Key Generation.* $\mathcal{K}(\kappa)$:

1. Select $z \in_R \mathbb{Z}_p$ as the secret key $\mathtt{sk}$
2. Set $Z := g^z$ as the public key $\mathtt{pk}$

*Signing.* $\mathcal{S}(m, \mathtt{sk})$:

1. Let $\mathtt{sk} = z$. Select $r \in_R \mathbb{Z}_p$, set $R := g^r$ and $c := \mathsf{H}(m, R)$.
2. The signature on $m$ is $\sigma := (y, R)$ where

$$y := r + zc$$

*Verification.* $\mathcal{V}(\sigma, m)$:

1. Let $\sigma = (y, R)$ and $c = \mathsf{H}(m, R)$.
2. $\sigma$ is valid if

$$g^y = RZ^c$$

# Security of Schnorr Signature–An Intuition

- Consider an adversary $\mathcal{A}$ with ability to launch chosen-message attack on the Schnorr signature scheme.
- Let $\{\sigma_0, \ldots, \sigma_{n-1}\}$ with $\sigma_i = (y_i = r_i + zc_i, R_i)$ on $m_i$ be the signatures that $\mathcal{A}$ receives.

$$\begin{pmatrix} 1 & 0 & \cdots & 0 & c_0 \\ 0 & 1 & \cdots & 0 & c_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & c_{n-1} \end{pmatrix} \times \begin{pmatrix} r_0 \\ r_1 \\ \vdots \\ r_{n-1} \\ z \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ r_{n-1} \end{pmatrix}$$

# Security of Schnorr Signature–An Intuition...

- *However*, $\mathcal{A}$ can solve for $x$ if it gets two equations containing the same $r$ but different $c$, i.e.

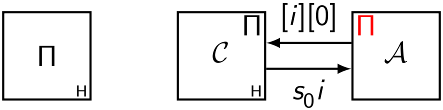
$$y = r + zc \quad \text{and} \quad \bar{y} = r + z\bar{c}$$

implies

$$z = \frac{y - \bar{y}}{c - \bar{c}} \sqcap$$

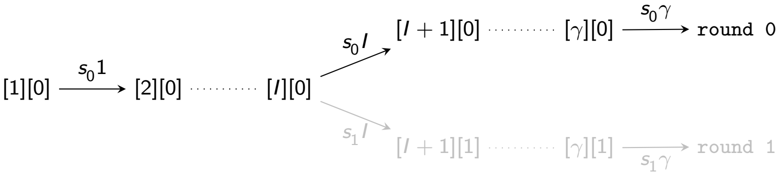
# The Oracle Replay Attack

- Random oracle H–$i^{th}$ random oracle query $[i][0]$ replied with $s_0 i$.




Tape re-wound to $[I][0]$

Simulation in `round 1` from $[I][0]$ using a *different* random function



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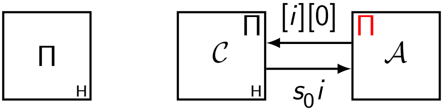


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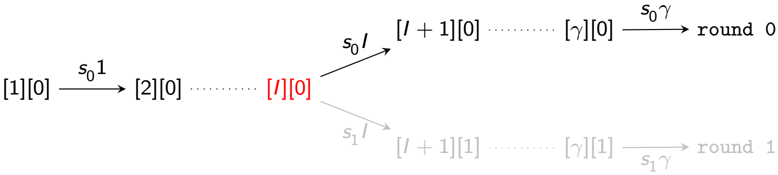

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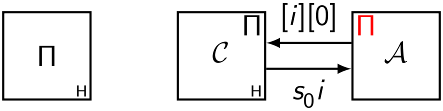


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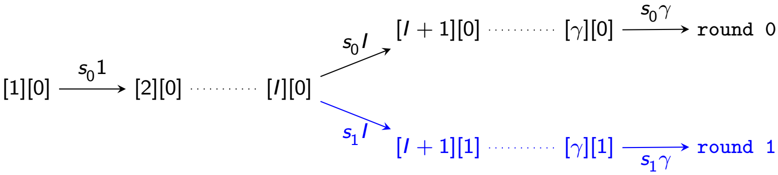
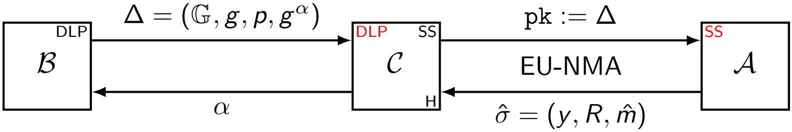
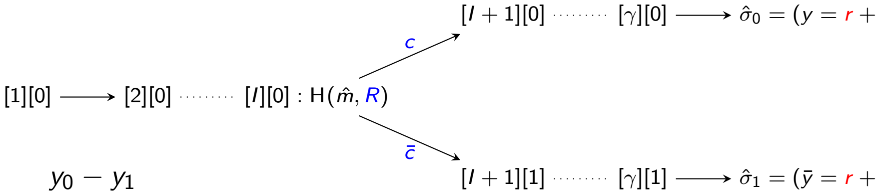


# Proving Security of Schnorr Signature using ORA




$$\alpha = \frac{y_0 - y_1}{c - \bar{c}}$$

# Forking Lemma

- The oracle replay attack formalised through the forking algorithm
- The *forking lemma* gives a lower bound on the success probability of the oracle replay attack ($frk$) in terms of the success probability of the adversary during a particular run ($acc$)

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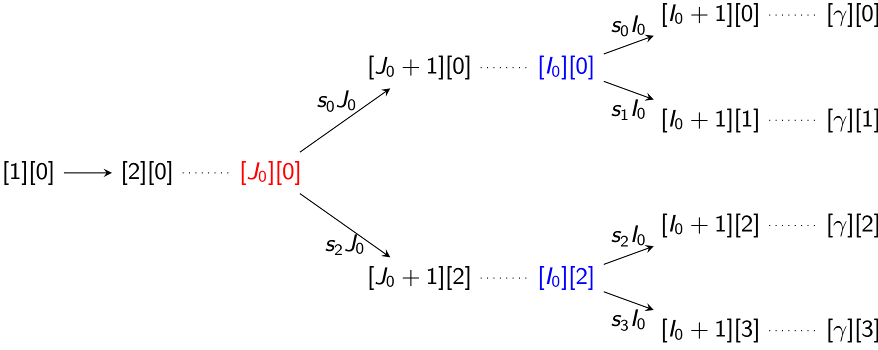
- The oracle replay attack formalised through the forking algorithm
- The *forking lemma* gives a lower bound on the success probability of the oracle replay attack (*frk*) in terms of the success probability of the adversary during a particular run (*acc*)
- Types of forking algorithms

| Forking Algorithm | #Oracles | #Replay Attacks | Success Prob. (≈) |
|---|---|---|---|
| GF–*General Forking* - $\mathcal{F}_{\mathcal{W}}$ | 1 | 1 (*i.e.* 2 runs) | $\frac{acc^2}{\gamma}$ |
| MF–*Multiple-Forking(n)* - $\mathcal{M}_{\mathcal{W},n}$ | 2 | 2n-1 (*i.e.* 2*n* runs) | $\frac{acc^n}{\gamma^{2n}}$ |

$\gamma$–Upper bound on the number of oracle queries

# Forking Lemma...

E.g. Multiple-forking algorithm for $n = 3$.



# GALINDO-GARCIA IBS–CONSTRUCTION

# The Construction

Set-up. $\mathcal{G}(\kappa)$:

1. Let $\mathbb{G} = \langle g \rangle$ be a group of prime order $p$.
2. Return $z\mathbb{Z}_p$ as msk and $(\mathbb{G}, p, g, g^z, \mathsf{H}, \mathsf{G})$ as mpk, where H and G are hash functions

$$\mathsf{H} : \{0,1\}^* \to \mathbb{Z}_p \text{ and } \mathsf{G} : \{0,1\}^* \to \mathbb{Z}_p.$$

# The Construction

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$$\mathsf{H} : \{0,1\}^* \to \mathbb{Z}_p \ \text{ and } \ \mathsf{G} : \{0,1\}^* \to \mathbb{Z}_p.$$

**Key Extraction.** $\mathcal{E}(\mathtt{id}, \mathtt{msk}, \mathtt{mpk})$:

1. Select $r\mathbb{Z}_p$ and set $R := g^r$.
2. Return $\mathtt{usk} := (y, R)$ as $\mathtt{usk}$, where

$$y := r + zc \ \text{ and } \ c := \mathsf{H}(R, \mathtt{id}).$$

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$$y := r + zc \quad \text{and} \quad c := \mathsf{H}(R, \mathtt{id}).$$

**Signing.** $\mathcal{S}(\mathtt{id}, m, \mathtt{usk}, \mathtt{mpk})$:

1. Let $\mathtt{usk} = (y, R)$. Select $a\mathbb{Z}_p$ and set $A := g^a$.
2. Return $\sigma := (A, b, R)$ as the signature, where

$$b := a + yd \quad \text{and} \quad d := \mathsf{G}(\mathtt{id}, A, m).$$

# The Construction

Verification. $\mathcal{V}(\sigma, \texttt{id}, m, \texttt{mpk})$:

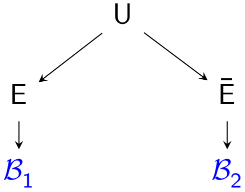
1. Let $\sigma = (A, b, R)$, $c := \mathsf{H}(R, \texttt{id})$ and $d := \mathsf{G}(\texttt{id}, A, m)$.
2. The signature is valid if

$$g^b = A(R \cdot (g^z)^c)^d.$$

# ORIGINAL SECURITY ARGUMENT

# Original Security Argument

- Let $\hat{\sigma} = (b, A, R)$ be the forgery produced by $\mathcal{A}$ on $(\hat{\mathtt{id}}, \hat{m})$.




E: Event that $\mathcal{A}$ forges using the same randomiser $R$ as given by $\mathcal{C}$ as part of signature query on $\hat{\mathtt{id}}$.

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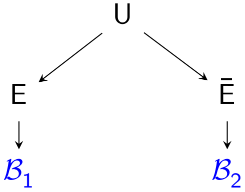


E: Event that $\mathcal{A}$ forges using the same randomiser $R$ as given by $\mathcal{C}$ as part of signature query on $\hat{\mathtt{id}}$.

- In both $\mathcal{B}_1$ and $\mathcal{B}_2$, solving DLP is *reduced* to breaking the IBS.

# In a Nutshell

| Reduction | Success Prob. (≈) | Forking Used |
|---|---|---|
| $\mathcal{B}_1$ | $\frac{\epsilon^2}{q_{\mathsf{G}}^3}$ | General Forking–$\mathcal{F}_{\mathcal{W}}$ |
| $\mathcal{B}_2$ | $\frac{\epsilon^4}{(q_{\mathsf{H}} q_{\mathsf{G}})^6}$ | Multiple-Forking–$\mathcal{M}_{\mathcal{W},3}$ |

# Our Contribution

- We found several problems with $\mathcal{B}_1$ and $\mathcal{B}_2$
  1. $\mathcal{B}_1$: Fails in the standard security model for IBS
  2. $\mathcal{B}_2$: All the adversarial strategies were not covered

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- The adversary *is able to* distinguish a simulation from the real execution of the protocol.

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  1. $\mathcal{B}_1$: Fails in the standard security model for IBS
  2. $\mathcal{B}_2$: All the adversarial strategies were not covered
- The adversary *is able to* distinguish a simulation from the real execution of the protocol.
- Positive contribution:
  1. We give a *detailed* new security argument
  2. *Tighter* than the original security argument

# NEW SECURITY ARGUMENT

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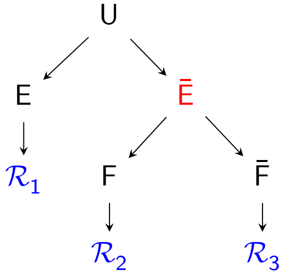


F: Event that $\mathcal{A}$ calls $\mathsf{G}(\hat{\mathtt{id}}, A, \hat{m})$ before $\mathsf{H}(R, \hat{\mathtt{id}})$.

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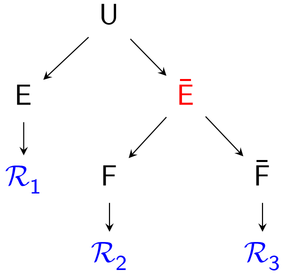
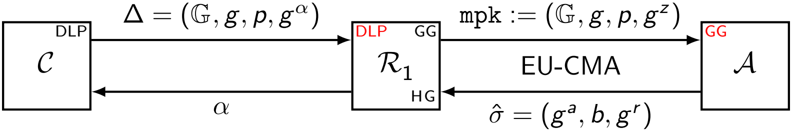



F: Event that $\mathcal{A}$ calls $\mathsf{G}(\hat{\mathtt{id}}, A, \hat{m})$ before $\mathsf{H}(R, \hat{\mathtt{id}})$.

1. Problems with $\mathcal{B}_1$ addressed in $\mathcal{R}_1$
2. $\mathcal{R}_2$ covers the unaddressed adversarial strategy in $\mathcal{B}_2$
3. $\mathcal{R}_3$ same as the original reduction $\mathcal{B}_2$

# Reduction $\mathcal{R}_1$




- Problem instance plugged in the randomiser $R$ (as in $\mathcal{B}_1$)

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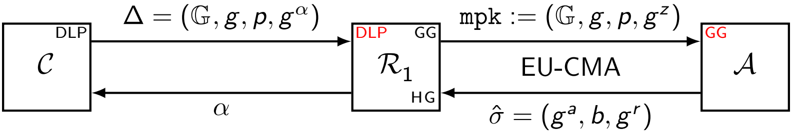
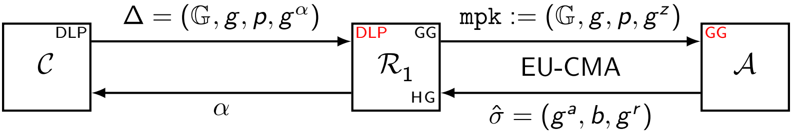



- Problem instance plugged in the randomiser $R$ (as in $\mathcal{B}_1$)
- *Coron's technique* used to assign target identities (instead of guessing) – security degradation *reduced* to $\mathrm{O}(q_\varepsilon)$
- Signature Query. $(\mathtt{id}, m)$ –
  - Toss a biased coin $\beta$

# Reduction $\mathcal{R}_1$




- Problem instance plugged in the randomiser $R$ (as in $\mathcal{B}_1$)
- *Coron's technique* used to assign target identities (instead of guessing) – security degradation *reduced* to $\mathrm{O}(q_\varepsilon)$
- Signature Query. $(\mathtt{id}, m)$ –
  - Toss a biased coin $\beta$
    1. If $\beta = 0$, signature given with randomiser $R$ containing $g^\alpha$
    2. Else, $\mathcal{R}_1$ uses knowledge of $\mathtt{msk}$ to generate user private key for $\mathtt{id}$ and then computes signature using $\mathcal{S}$

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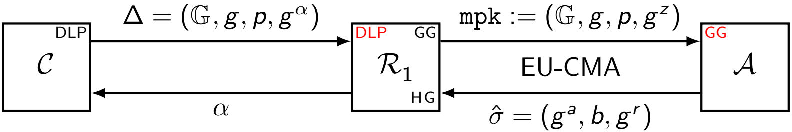


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- *General forking algorithm* $(\mathcal{F}_{\mathcal{W}})$ used to solve DLP (as in $\mathcal{B}_1$)

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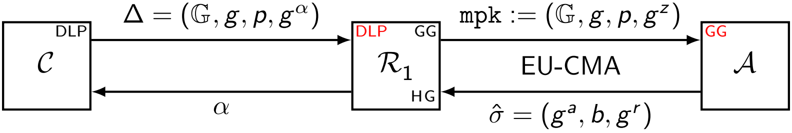
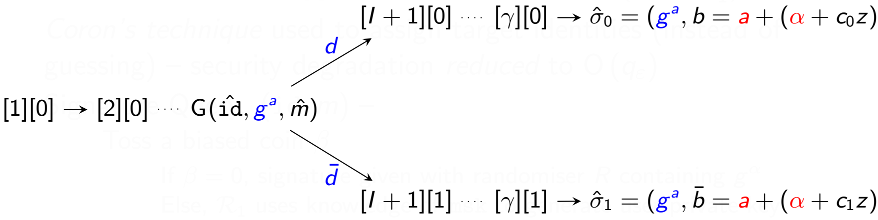
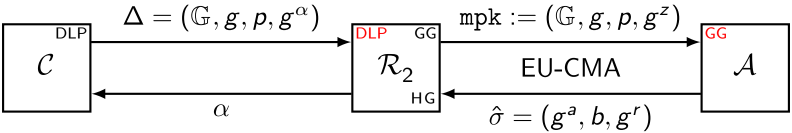


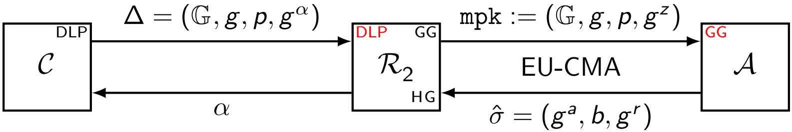



# Reduction $\mathcal{R}_2$

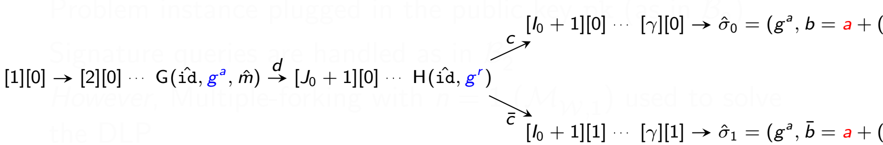



- Problem instance plugged in the public key $\texttt{pk}$ (as in $\mathcal{B}_2$)
- Signature queries are handled as in $\mathcal{B}_2$
- *However*, Multiple-forking with $n = 1$ ($\mathcal{M}_{\mathcal{W},1}$) used to solve the DLP
- Hence, tighter than $\mathcal{B}_2$

# Reduction $\mathcal{R}_2$




- Problem instance plugged in the public key pk (as in $\mathcal{B}_2$)
- Signature queries are handled as in $\mathcal{B}_2$
- *However*, Multiple-forking with $n = 3$ ($\mathcal{M}_{\mathcal{W},1}$) used to solve the DLP
- Hence, *tighter* than $\mathcal{B}_2$



# In a Nutshell

| Reduction | Success Prob. ($\approx$) | Forking Used |
|---|---|---|
| $\mathcal{R}_1$ | $\frac{\epsilon^2}{q_{\mathsf{G}} q_{\varepsilon}}$ | $\mathcal{F}_{\mathcal{W}}$ |
| $\mathcal{R}_2$ | $\frac{\epsilon^2}{(q_{\mathsf{H}}+q_{\mathsf{G}})^2}$ | $\mathcal{M}_{\mathcal{W},1}$ |
| $\mathcal{R}_3$ | $\frac{\epsilon^4}{(q_{\mathsf{H}}+q_{\mathsf{G}})^6}$ | $\mathcal{M}_{\mathcal{W},3}$ |

# Conclusion and Future Work

We revisited the Galindo-Garcia IBS security argument

- Analysed the original security proof; fixed ambiguities
- Provided an improved security proof

Future Work

- Replacing the 'costly' multiple-forking for even tighter reductions–*dependent* random oracles.

# THANK YOU!